THEORETICAL AND COMPARATIVE STUDY REGARDING THE MECHANICS DISPLASCEMENTS UNDER THE STATIC LOADINGS FOR THE SQUARE PLATE MADE BY WOOD REFUSE AND MASSIF WOOD

Marius Şerban Fetea, Gabriel Cheregi*

*University of Oradea, Faculty of Environmental Protection, 26 Gen. Magheru St., 410048 Oradea; Romania

Abstract

In this comparative study the both plate have the same rectangular surface $a \times b$ *and thickness* g. *This paper represents a theoretical and comparative study made it to obtain the approximate roots for the static displacement.*

Key words: Square, refuse, massif plate, simply supported, method, displacement.

INTRODUCTION

For the plate made by the refuse wood the calculus have been made similarly which a isotropic plate using the variational Ritz method. The both plates have simply supported all the edges and are loading with a uniformly forces having the intensity q.

The Poisson coefficients is for the both plate $\mu = 0$

The Ritz methods consist in selecting a suitable infinite series expression of the deflection which satisfies the geometrical boundary conditions, and satisfaction of the differential equation motion is not required.

We use for the deflection expression:

$$w(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot X_i(x) \cdot Y_j(y)$$

Where:

- $c_{ij}, X_i(x), Y_j(y)$ – represents the unknown coefficients obtained from the minimum total energy principle and the appropriate displacement functions which individually satisfy at least geometrical boundary conditions.

MATERIALS AND METHODS

We consider two rectangular plates, having the dimensions $a \times b = 1000 \times 1000 \left[mm^2\right]$, loaded by a uniformly forces having the weight intensity $q.$.

The initials values for the *massif wood* plate are:

$$E = (0{,}1 - 0{,}12) \times 10^6 \left[\frac{daN}{cm^2}\right]$$

$$G = 0{,}055 \times 10^5 \left[\frac{daN}{cm^2}\right]$$

$$\mu = 0$$

$$g = 0{,}25\left[cm\right]$$

The initials values for the *wood refuse* plate are:

$$E = 0{,}28 \times 10^5 \left[\frac{daN}{cm^2}\right]$$

$$\mu = 0$$

$$g = 0{,}25\left[cm\right]$$

Where:

$E -$ Young's modulus.

$g -$ thickness.

$\mu -$ the Poisson coefficients.

The boundary conditions for the plates with all sides simply supported are:

$$w = 0, M_x = 0 \text{ for } x = 0, a$$

$$w = 0, M_y = 0 \text{ for } y = 0, b$$

The variational Ritz methods consist in application of the minimum potential energy theory.

$\Pi -$ the total potential energy of the plate.

We use for deflection the approximate expression:

$$w(x, y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot f_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot X_i(x) \cdot Y_j(y)$$

$$f_{ij}(x, y) = X_i(x) \cdot Y_j(y)$$

Where:

$X_i(x), Y_j(y)$ – Represents functions who satisfies the geometrical boundary conditions and closely approximates the shape.

For the plates with all sides simply supported the shape functions have been choose like a product between two trigonometrically functions having distinct variables.

We represents the shape functions as an double infinite series:

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b}$$

Using the minimum energy method it can be determinated the unknown coefficients $c_{ij.}$

Solving a linear algebric system obtain from the minimum conditions $\Pi = 0$ result the coefficients $c_{ij.}$

$\Pi = \min$

$$\frac{\partial \Pi}{\partial c_{ij}} = 0$$

Solutions for the massif wood plate and refuse wood plate

The deformation energy is given by the relation between total energy and mechanical work:

$$W = \Pi - U$$

$W -$ strain energy.

$U -$ Kinetic energy.

$$W = \frac{D}{2}\int_A \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 dxdy$$

$$W = \frac{\pi^4 D}{2}\int_0^a\int_0^b \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right]^2 dxdy$$

The first two derivates of deflection given by the variables x, y are:\

$$\frac{\partial w(x,y)}{\partial x} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{i\pi}{a} \cdot \cos\frac{i \cdot \pi \cdot x}{a} \cdot \sin\frac{j \cdot \pi \cdot y}{b}$$

$$\frac{\partial^2 w(x,y)}{\partial x^2} = -\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{i^2\pi^2}{a^2} \cdot \sin\frac{i \cdot \pi \cdot x}{a} \cdot \sin\frac{j \cdot \pi \cdot y}{b}$$

$$\frac{\partial w(x,y)}{\partial y} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{j\pi}{b} \cdot \sin\frac{i \cdot \pi \cdot x}{a} \cdot \cos\frac{j \cdot \pi \cdot y}{b}$$

$$\frac{\partial^2 w(x,y)}{\partial x^2} = -\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{j^2\pi^2}{b^2} \cdot \sin\frac{i\cdot\pi\cdot x}{a} \cdot \sin\frac{j\cdot\pi\cdot y}{b}$$

With the assumption that:

$$\sin\frac{i\pi x}{a} \cdot \sin\frac{k\pi x}{a} = \begin{cases} 0, i \neq k \\ \frac{a}{2}, i = k \end{cases}$$

$$\sin\frac{j\pi y}{b} \cdot \sin\frac{k\pi y}{b} = \begin{cases} 0, i \neq k \\ \frac{b}{2}, i = k \end{cases},$$

the plate deformation energy expression is:

$$W = \frac{\pi^4 abD}{8}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)$$

$$U = \int_A q(x,y)w(x,y)dxdy =$$

$$= \int_A \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right] \cdot q(x,y)dxdy =$$

$$= q(x,y) \cdot \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \int_0^a\int_0^b \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}dxdy =$$

$$= q(x,y) \cdot \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \int_0^a \sin\frac{i\pi x}{a}dx \cdot \int_0^b \sin\frac{j\pi y}{b}dy$$

For:

$$i = 1,3,5,...$$
$$j = 1,3,5,...,$$

the expression for mechanical work is:

$$U = \frac{4\cdot q(x,y)ab}{\pi^2}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \frac{1}{i\cdot j}$$

Replacing into the energy expression we obtain:

$$\Pi = \frac{\pi^4 ab\cdot D}{8} \cdot \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right) - \frac{4\cdot q(x,y)ab}{\pi^2}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \frac{1}{i\cdot j}$$

$D-$ the plate rigidity .

$$D = \frac{E\cdot g^3}{12(1-\mu^2)}$$

Using the conditions $\frac{\partial \Pi}{\partial c_{ij}} = 0$, results the equations system:

$$\frac{\pi^4 ab \cdot D}{8} \cdot 2c_{ij}\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right) - \frac{4 \cdot q(x,y)ab}{\pi^2} \cdot \frac{1}{i \cdot j} = 0$$

$$c_{ij} = \frac{16q(x,y)}{\pi^6 D \cdot \left[\frac{1}{i \cdot j\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^2}\right]}$$

For massif wood plate the plate rigidity is:

$$D = 0{,}15625 \times 10^9 [daN \cdot cm],$$

and for the wood refuse plate $D = 0{,}36458 \times 10^8 [daN \cdot cm]$

The calculus for the maximum deflection values in the middle of the plates was made for $i = 1$ and $j = 1$ at:

$$x = 50[cm]$$

$$y = 50[cm]$$

$$w(x, y = \frac{16q(x,y)}{\pi^6 D} \cdot \sum_{i=1}^{m} \sum_{j=1}^{n} \cdot \frac{\sin\frac{i\pi x}{a} \cdot \sin\frac{j\pi y}{b}}{\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^2}$$

The both plates are loaded by a uniformly forces having the intensity q. Also the weight work like a uniformly forces on the volume of each plate.

$w_1^{'}$ – refuse wood plate deflection.

$w_2^{'}$ – massif wood plate deflection.

$\rho_1^{'}$ – refuse wood density.

$\rho_2^{'}$ – massif wood density.

$$\rho_1^{'} = 600\left[\frac{Kg}{m^3}\right]$$

$$\rho_2^{'} = 900\left[\frac{Kg}{m^3}\right]$$

In concordance with superposition principle the deflections for each plates having uniformly loading are:

$$w_1' = 0{,}8[cm]$$
$$w_2' = 0{,}2[cm]$$

DISCUSSIONS AND CONCLUSIONS

We obtain for the displacement in the center of the refuse wood plate an admitted value which it can be calculated the efforts and the tension from any cross section of the plate.

Using the plate made by refuse wood the conclusion are:

- the plates can be use for the light constructions to achieve the floors.
- reduce the weight of the construction;
- reduce the price of building;
- can be achieve in a lot of size and have an economic efficient.

Having the maximum values for deflections in the middle of the plates can be establish the dangerous sections and also the sectionals efforts (bending moments).

This comparative studies represents a mathematics mode to solve the *plate* deflections and give us some information for them who help us to choose the adequate materials and the accepted loading thus to resists without the possibility of the brake.

REFERENCES

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