THEORETICAL STUDY REGARDING THE FREE VIBRATIONS OF SIMPLY SUPPORTED FLAT PLATE

Marius Serban Fetea* Mariana Vlad **

University of Oradea, Faculty of Environmental Protection, 26 Gen. Magheru St., 410048 Oradea; Romania

Abstract
The rectangular flat plates, as well as the the angular ones in general, often intervene as strength elements in the structures of civil and industrial constructions, their actual shape and support mode being imposed by different conditions in the exploitation of the buildings, such as the lay-out of some technological appliances, pipe crossing, embrasures in the stairs and so on.
Determining the solution of the plate's differential equation with partial derivatives that satisfies all the kinematic and static conditions on the considered boundary cannot always be rigurously achieved. As a result, the variational methods represent in many cases, an effective tool in obtaining the solution of the differential or partial derivative equation that can rigurously satisfy the differential equation and partially the static boundary conditions.
Solving the differential equations using the variational methods consists in the replacement of the unknown function that satisfies both the differential equation and the boundary conditions, with an approximate analytical expression chosen in such a way as to approximate the sought out function as well as possible, meaning that the deviation from the real value of the function should be minimum.

Key words: square plate, simply supported, variational method, mode shapes, eigenfunctions.

INTRODUCTION

As for the historic record of the problem related to the study of flat plates, the first results were out for publishing at the end of the 18th century, the beginning of the 19th century, having Chladni E, Strehlke, Konig, R, Tanaka S, Rayleigh L, Ritz W and later on Gontkevich V, Timoshenko S, Leissa as pioneers. Each of the above mentioned authors have had significant contributions regarding the development of methods in order to solve the plates and establish some rigurous solutions of their differential equations of equilibrium.

The making of constructions, machines and different high-perfomance appliances, whose functioning should take place in safety conditions, have required theoretical studies of rich complexity, as well as practical experiments, within which the problem of their free and forced vibrations represent an important category in the respective theme of research. The importance of studying the vibrations of different deformable material systems (elastic systems in constructions, technological equipments, mobile or stationary machines and equipments), whose structures take in types of plates different in terms of shape, loading mode and boundary conditions characterised by forced or free vibration motions and carried on to the structure itself, has been made obvious by the system's degradation in time.

By means of the dynamic analysis of plates there has been an emphasis on the complexity of the notion of dynamic calculus, which has the following as main working stages:

- establishing the dynamic model considered for the first time

- determining the normal vibration modes (self pulsations and vectors, respectively the functions of vibration modes)
- determining the dynamic response in displacements and sectional stresses
- checking stability and strength conditions

MATERIALS AND METHODS

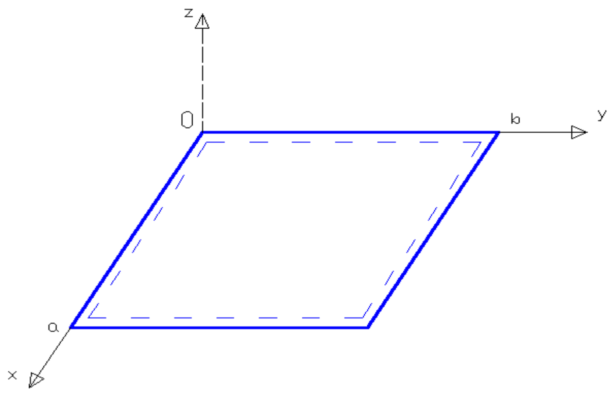

Figure1.

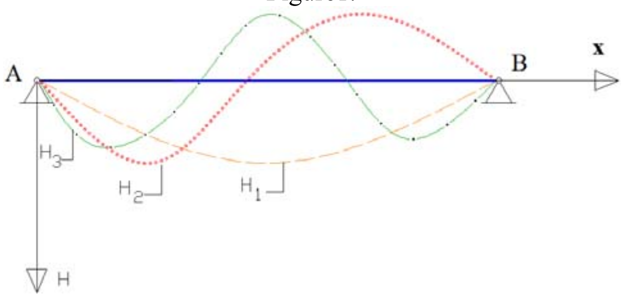

Figure 2.

The object of study is determining the vibration modes, eigenvalues ω_{ij} (represented by pulsation parameter $\sqrt{\lambda_{ij}}$) and the eigenfunctions $\Phi_{ij}(x,y)$ .There is a unitary presentation of the calculus algorithm of the variational method Galerkin-Vlasov for a simply supported plate *figure 1* having boundary conditions and different edge ratios $\alpha = a/b$.

In determining these normal vibration modes, the variational method Galerkin-Vlasov is adapted, being regarded as a particularity of the Bubnov-Galerkin method for the dynamic infinite-dimensional systems.

The study of the rectangular plate started from the normal vibration mode equation of the plates, that expresses their dynamic equilibrium, choosing for the plate functions the products of the shape functions of the beams with the same boundary conditions as the plate in x and respectively y direction *figure 2.*

The equation of the normal vibration modes

$$\nabla^4 \Phi_{ij}(x,y) = \lambda_{ij} \Phi_{ij}(x,y)$$

$$\Phi_{ij}(x,y) = X_i(x) \cdot Y_j(y)$$

in which ∇^4 is the double Laplacean operator, together with the boundary conditions, represents a Sturm-Liouville problem, whose solving with the suggested method leads to the charateristics of pulsations and vibration shapes.

The equation of the normal vibration modes, along with the homogeneous boundary conditions describe a type Sturm-Liouville problem, [3], [4]. Solving type Sturm –Liuoville problem allows the determination of inherent values λ_{ij}, and the functions of the vibration shapes of beams $y_i = H_i(x)$. Knowing the inherent values λ_{ij}, gives pulsations eigenvalues ω_i, for i =1,2,3,.. ∞.

The expression of the parameter of specific pulsations obtained by applying the suggested method is

$$\lambda_{ij} = \frac{\int_0^a X_i^{IV}(x)\cdot X_i(x)dx\cdot\int_0^b Y_j^2(y)dy + 2\cdot\int_0^a X_i^{''}(x)\cdot X_i(x)dx\cdot\int_0^b Y_j^{''}(y)\cdot Y_j(y)dy + \int_0^a X_i^2(x)dx\cdot\int_0^b Y_j^{IV}(y)\cdot Y_j(y)dy}{\int_0^a X_i^2(x)dx\cdot\int_0^b Y_j^2(y)dy},$$

in which $X_i(x)$ şi $Y_j(y)$, are the eigenfunctions of the beams that have the same boundary conditions as the plate on directions x respectively y. Knowing the inherent values λ_i gives self-pulsations plate (eigenvalues) values of ω_i

$$\omega_{ij} = \frac{1}{a^2}\sqrt{\lambda_{ij}}\sqrt{\frac{D}{\rho}}$$

The use of Galerkin-Vlasov method for determining the normal vibration modes of the plates is reduced to the evaluation of the integrals defined above.

For the studied rectangular flat plate simply supported on the boundary, the pulsation parameters for a number of 3 normal vibration modes are determined. The obtained results regarding the pulsation parameters for the studied flat rectangular plate are subsequently presented.

Rectangular flat plate simply supported on the boundary

Using the suggested method, we determine the values pulsation parameters adequate to the normal vibration modes (1,1), (2,1), (3,1) and the plate edge ratios, $\alpha = a/b = 1$, $\alpha = a/b = 1{,}5$, $\alpha = a/b = 2$. The values for the pulsation parameters are presented in *table 1*.

Table 1.

Vibration Mode	Mode (1,1)	Mode (2,1)	Mode (3,1)
$\sqrt{\lambda_{ij}}, \alpha = 1$	19.7392	49,34	78,65
$\sqrt{\lambda_{ij}}, \alpha = 1{,}5$	32,07	61,68	111.03
$\sqrt{\lambda_{ij}}, \alpha = 2$	49,34	78,95	128,3

Also, are presented the eigenfunctions of the plate having the same boundary conditions as the beams on directions x respectively y and 3 modes shapes. The eigenfunctions are presented in *tables 2, 3, 4* and the 3 mode shapes in *figures no. 3, 4, 5*. The calculus was made by the author using Matlab 6.0.

Table 2.

		Eigenfunctions							Mode (1,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,095492	0,181636	0,25	0,293893	0,309017	0,293893	0,25	0,181636	0,095491	-1,4E-08
0,8	0	0,181636	0,345492	0,475528	0,559017	0,587785	0,559017	0,475528	0,345491	0,181636	-2,7E-08
1,2	0	0,25	0,475528	0,654509	0,769421	0,809017	0,769421	0,654508	0,475528	0,25	-3,8E-08
1,6	0	0,293893	0,559017	0,769421	0,904509	0,951057	0,904508	0,769421	0,559017	0,293893	-4,4E-08
2	0	0,309017	0,587785	0,809017	0,951057	1	0,951057	0,809017	0,587785	0,309017	-4,6E-08
2,4	0	0,293893	0,559017	0,769421	0,904508	0,951057	0,904508	0,769421	0,559017	0,293893	-4,4E-08
2,8	0	0,25	0,475528	0,654508	0,769421	0,809017	0,769421	0,654508	0,475528	0,25	-3,8E-08
3,2	0	0,181636	0,345491	0,475528	0,559017	0,587785	0,559017	0,475528	0,345491	0,181636	-2,7E-08
3,6	0	0,095491	0,181636	0,25	0,293893	0,309017	0,293893	0,25	0,181636	0,095491	-1,4E-08
4	0	-1,4E-08	-2,7E-08	-3,8E-08	-4,4E-08	-4,6E-08	-4,4E-08	-3,8E-08	-2,7E-08	-1,4E-08	2,15E-15

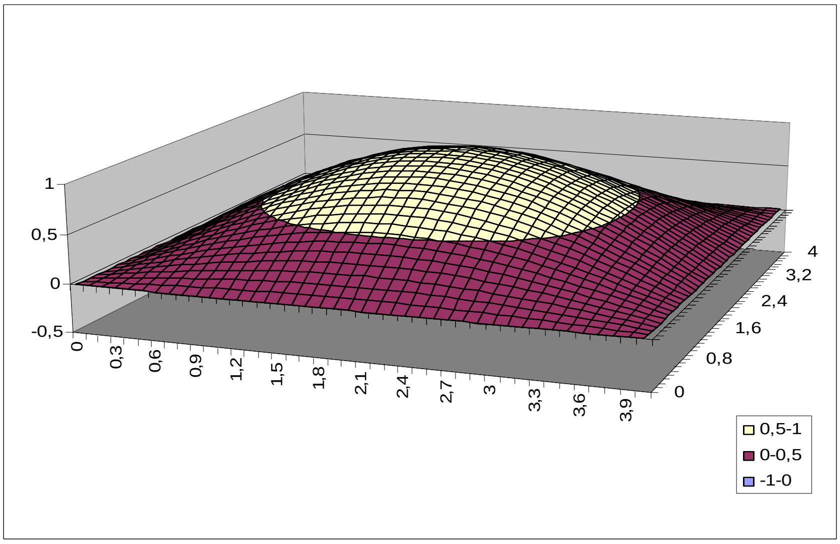

Figure 3.

Table 3.

		Eigenfunctions Mode (2,1)									
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,181636	0,345492	0,475528	0,559017	0,587785	0,559017	0,475528	0,345491	0,181636	-2,7E-08
0,8	0	0,293893	0,559017	0,769421	0,904509	0,951057	0,904508	0,769421	0,559017	0,293893	-4,4E-08
1,2	0	0,293893	0,559017	0,769421	0,904508	0,951057	0,904508	0,769421	0,559017	0,293893	-4,4E-08
1,6	0	0,181636	0,345491	0,475528	0,559017	0,587785	0,559017	0,475528	0,345491	0,181636	-2,7E-08
2	0	-1,4E-08	-2,7E-08	-3,8E-08	-4,4E-08	-4,6E-08	-4,4E-08	-3,8E-08	-2,7E-08	-1,4E-08	2,15E-15
2,4	0	-0,18164	-0,34549	-0,47553	-0,55902	-0,58779	-0,55902	-0,47553	-0,34549	-0,18164	2,73E-08
2,8	0	-0,29389	-0,55902	-0,76942	-0,90451	-0,95106	-0,90451	-0,76942	-0,55902	-0,29389	4,41E-08
3,2	0	-0,29389	-0,55902	-0,76942	-0,90451	-0,95106	-0,90451	-0,76942	-0,55902	-0,29389	4,41E-08
3,6	0	-0,18164	-0,34549	-0,47553	-0,55902	-0,58779	-0,55902	-0,47553	-0,34549	-0,18164	2,73E-08
4	0	2,87E-08	5,46E-08	7,51E-08	8,83E-08	9,28E-08	8,83E-08	7,51E-08	5,46E-08	2,87E-08	-4,3E-15

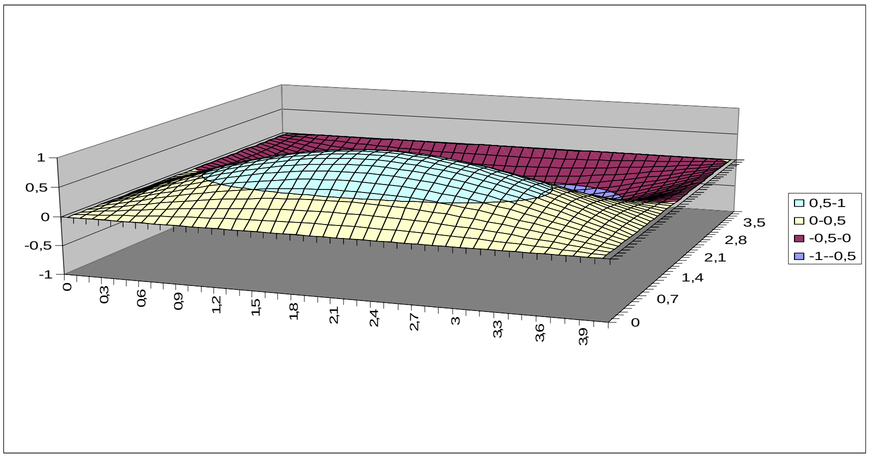

Figure 4.

Table 4.

		Eigenfunctions							Mode (3,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,25	0,475528	0,654509	0,769421	0,809017	0,769421	0,654508	0,475528	0,25	-3,8E-08
0,8	0	0,293893	0,559017	0,769421	0,904508	0,951057	0,904508	0,769421	0,559017	0,293893	-4,4E-08
1,2	0	0,095491	0,181636	0,25	0,293893	0,309017	0,293893	0,25	0,181636	0,095491	-1,4E-08
1,6	0	-0,18164	-0,34549	-0,47553	-0,55902	-0,58779	-0,55902	-0,47553	-0,34549	-0,18164	2,73E-08
2	0	-0,30902	-0,58779	-0,80902	-0,95106	-1	-0,95106	-0,80902	-0,58779	-0,30902	4,64E-08
2,4	0	-0,18164	-0,34549	-0,47553	-0,55902	-0,58779	-0,55902	-0,47553	-0,34549	-0,18164	2,73E-08
2,8	0	0,095492	0,181636	0,25	0,293893	0,309017	0,293893	0,25	0,181636	0,095492	-1,4E-08
3,2	0	0,293893	0,559017	0,769421	0,904509	0,951057	0,904509	0,769421	0,559017	0,293893	-4,4E-08
3,6	0	0,25	0,475528	0,654508	0,769421	0,809017	0,769421	0,654508	0,475528	0,25	-3,8E-08
4	0	-4,3E-08	-8,2E-08	-1,1E-07	-1,3E-07	-1,4E-07	-1,3E-07	-1,1E-07	-8,2E-08	-4,3E-08	6,46E-15

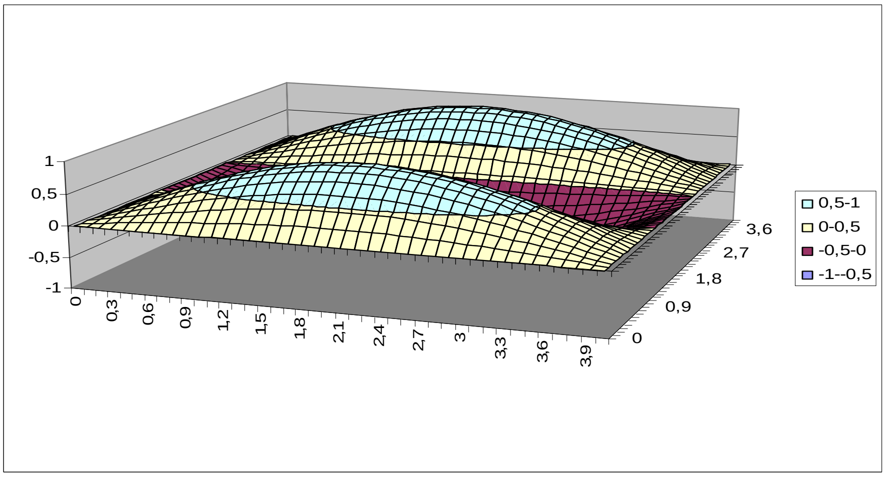

Figure 5.

DISCUSSION AND CONCLUSION

To validate the suggested method, we compare the achieved theoretical results with the ones determined by other authors when applying the Levy method, Rayleigh method and finite element method..

Ther is a presentation of the percentage deviations of the vibration parameters determined by applying the suggested method in comparison to the ones determined by other authors through the use of the analytical, variational and numerical methods.

Table 5.

Pulsation parameters	$\sqrt{\lambda_{11}}$	$\sqrt{\lambda_{21}}$	$\sqrt{\lambda_{31}}$
Metoda propusă	19.7392	49,34	78,65
Polidor Bratu [4] Levy Method	19,74 0,06 %	49,4 0,13 %	79,05 0,51 %
Szilard[5] Rayleigh Method	19,722 0,08 %	50,6 2,5 %	-
Szilard[5] F.E.M.	19,73 0 %	49,34 0 %	78,95 0,31 %
Bârsan [4]	19,739 0 %	49,350 0,03 %	78,973 0,41 %
Janich [3] Rayleigh Method	19,71 0,14 %	-	-

From the analysis of present data, we note that for the rectangular flat plate considered in this paper, the percentage deviations are in the limits of high precision.

For the rectangular flat plate simply supported on the boundary, the values of the pulsation parameters determined by the suggested method are compared with the values obtained by Szilard [5], when using the finite element method, respectively by Polidor Bratu[4], who applied the trigonometric series method to solve the plate.

The fundamental pulsation parameters of the flat square plate determined with the suggested method is equal to the one determined by Polidor Bratu [4], when applying the trigonometric series method and by Szilard [4], who applied the finite element method.

For the other normal vibration modes considered, that is (2,1), (3,1), the percenatge deviations of the pulsation parameteres determined by means of appplying the suggested method are small, in comparison to the ones achieved by the authors in [4] and [5]: between a minimum value of 0% and a maximum value of 2,5% for vibration mode (2,1), respectively a minimum deviation of 0,31 % and a maximum one of 0,51 % for vibration mode (3,1). By means of a clear, ordered and logical structuring of its chapters, the paper tries to present the dynamic analysis of rectangular simply supported flat plate subjected to free vibrations.

The content of the paper has been conceived and achieved in such a way as to emphasize the essential theoretical aspects, along with the adequate physical and mathematical subtilities and the actual problem in the practice of dynamic analysis of rectangular flat plate. The paper contains not only information from Romanian and worldwide field literature, supported and represented by illustrious proffesors and researchers of the dynamic school in our country, but also personal contributions regarding the determined dynamic characterisitcs that can be a valuable data base within subsequent research.

From the study of field literature it has been noted that there is no data concerning the results obtained by other authors regarding the values of the shape functions, respectively the parameters of the rectangular plate pulsations and the pulsations proper by means of applying the Galerkin-Vlasov variational method, a reason why the results presented in the paper represent a novelty element brought by the author.

REFERENCES

1. Bârsan, G., M.,1979, Structural dynamics and stability, Editura Didactică şi Pedagogică Bucureşti.
2. Bors I, 2007, Aplicatii ale problemei de valori proprii in mecanica constructiilor, Editura U.T.Pres. Cluj-Napoca.
3. Janich, R., Die naherungsweise Berechnung der Eigenfrequenzen von rechteckigen Platten bei verschiedenen.
4. Polidor, B., 2000, The Vibration of Elastic Systems, Editura Tehnică, Bucureşti.
5. Szilard, R., 1974, Theory and Analysis of Plates, Prentice-HallInc., Englewood Cliffs, New Jersey