THE FREE VIBRATIONS OF SQUARE PLATE MADE BY WOOD, CLAMPED ON TWO OPPOSITE SIDES, SIMPLY SUPPORTED AND FREE ON THE OTHERS, USING VARIATIONAL GALERKIN-VLASOV METHOD.

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Abstract

In engineering practice, plate problems often involve consideration of dynamic disturbances produced by time-dependent external forces or displacements. The undamped free flexural vibrations of rectangular plates are basically boundary value problems of the mathematical physics. For this rectangular plate made by wood, we determine the values of the specific pulsations adequate to the first 3 normal vibration modes. The study of this rectangular plate started from the normal vibration mode equation of the plates and represents the dynamic equilibrium.

Key words: plate, Galerkin-Vlasov, clamped,simply supported, free, edge variation method.

INTRODUCTION

The rectangular flat plates, as well as the angular ones in general, often intervene as strength elements in the structures of civil and industrial constructions, their actual shape and support mode being imposed by different conditions in the exploitation of the buildings, such as the lay-out of some technological appliances, pipe crossing, embrasures in the stairs and so on.

The results of the scientific research regarding the flat plates, as well as the practical importance related to the knowledge of their way of behavior in different loading and support situations within some structures (machines, buildings, equipments) are emphasized in numerous treaties, books, scientific papers published throughout the centuries.

The object of the thesis consists of the study of the free vibration of rectangular flat plates with different support conditions. The analyzed flat plates are thin, elastic and isotropic with stiffness at bending and meet the availability conditions of Kirchhoff hypotheses.

The suggested calculus method is an adaptation of the variation method Galerkin-Vlasov from the static calculus of dynamic plates and has been elaborated in such a way that the calculi necessary for determining the dynamic characteristics of the plates are made on the basis of some programs designed by the author himself.

MATERIALS AND METHODS

Bors (2007), Fetea (2010) and Szillard (1974) determinate the solution in the case of freely vibrating plates reduces to a homogeneous differential equations

$$\nabla^4 w(x,y,t) + \frac{\rho h}{D}\ddot{w}(x,y,t) = 0,$$

Where: x and y are Cartesian coordinates in the plane of the middle surface, ρ density, w displacements and t time.

For the case of freely vibrating plate, the external force is zero and the effects of the rotational inertia forces are neglected. We assume the solutions of equation in the form:

$$w(x,y,t)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\Phi_{ij}(x,y)\cdot\eta_{ij}(t)=\sum\sum\Phi_{ij}(x,y)\cdot A_{ij}\cdot\sin(\omega_{ij}t-\varphi_{ij}),$$

Where:

$$\Phi_{ij}(x,y)=X_i(x)\cdot Y_j(y)$$

Represents the shape function of the vibration, while the time-dependency of the displacements $\eta(t)$, are assumed to be harmonic. The solution, $w(x,y,t)$, must satisfy the boundary conditions of the plate and the initial conditions of the motion at t = 0; these conditions are $(w)_{t=0}=(\dot{w})_{t=0}=0$. $Y_j(y)$, represents the *i*th mode of a freely vibrating uniform beam, with length *a* and $Y_j(y)$ is the *j*th mode of a beam of length b.

The equation of the normal free vibration modes

$$\nabla^4\Phi(x,y)=\lambda\Phi(x,y),$$

in which: ∇^4 is the double Laplacean operator, together with the boundary conditions, represents a Sturm-Liouville problem, whose solving with the suggested method leads to the charateristics of pulsations and vibration shapes. By applying the variation Galerkin-Vlasov method, the problem of free flexural vibration of plates can be reduced to the solution of the following variation equations:

$$\nabla^4\Phi_{ij}(x,y)=\lambda_{ij}\cdot\Phi_{ij}(x,y),$$

$$\iint_A\left[\nabla^4\Phi_{ij}(x,y)-\frac{\rho\cdot\omega_{ij}^{\ 2}}{D}\Phi_{ij}(x,y)\right]\cdot\Phi_{ij}(x,y)dxdy=0.$$

By introducing the notations:

$$I_1=\iint_A[\Phi_{ij}(x,y)\cdot\nabla^4\Phi_{ij}(x,y)]dxdy,$$

$$I_2=\iint_A\Phi_{ij}^2(x,y)dxdy,$$

An approximate analytical expression for the pulsations of the free flexural vibration of plates with uniform thickness is:

$$\omega_{ij}^2=\frac{I_1}{I_2}\frac{D}{\rho}=\lambda_{ij}\frac{D}{\rho}.$$

Where: *i*, *j*, can takes the values 1, 2, 3, ...

Introducing the ∇^4 the double Laplace an operator, in equation and separate the independent variables x and y, we obtained for the expression of the parameter of specific pulsations

$$X_i^{''''}(x)\cdot Y_j(y)+2\cdot X_i^{''}(x)\cdot Y_j^{''}(y)+X_i(x)\cdot Y_j^{''''}(y)=\lambda_{ij}\cdot X_i(x)\cdot Y_j(y)$$

$$\lambda_{ij}=\frac{\int_0^a X_i^{''''}(x)\cdot X_i(x)dx\cdot\int_0^b Y_j^2(y)dy+2\cdot\int_0^a X_i^{''}(x)X_i(x)dx\cdot\int_0^b Y_j^{''}(y)\cdot Y_j(y)dy+\int_0^a X_i^2dx\cdot\int_0^b Y_j^{''''}\cdot Y_j(y)dy}{\int_0^a X_i^2(x)dx\cdot\int_0^b Y_j^2(y)dy}$$

The pulsations of plate can be calculated using the expression

RESULTS AND DISCUSSION

The normal vibration modes for the combinations of conditions at the ends of the beam are determined. The following cases are studied:

- doubly-clamped beam, having the shape function $y_i = G_i(x)$,
- hinged -free beam, having the shape function $y_i = F_i(x)$.

Considering the variation method Galerkin-Vlasov, we determining the vibration modes, self – pulsations ω_{ij} (represented by pulsation parameter $\sqrt{\lambda_{ij}}$) and the vibration functions $\Phi_{ij}(x,y)$

$$G_i(x)=\left(\cosh\beta_i\frac{x}{a}-\cos\beta_i\frac{x}{a}\right)-k_i\cdot\left(\sinh\beta_i\frac{x}{a}-\sin\beta_i\frac{x}{a}\right)$$

$$F_1(y)=\sqrt{3}\cdot\frac{y}{b}$$

$$F_j(y)=\left(\sinh\beta_j\frac{y}{b}+\sin\beta_j\frac{y}{b}\right)-k_j\cdot\left(\sinh\beta_j\frac{y}{b}-\sin\beta_j\frac{y}{b}\right)$$

$$\Phi_{ij}(x,y)=G_i(x)\cdot F_j(y).$$

$$[(G_i^{''''}(x)\cdot F_j(y)+2\cdot G_i^{''}(x)\cdot F_j^{''}(y)+G_i(x)\cdot F_j^{''''}(y)]\cdot G_i(x)\cdot F_j(y)=\lambda_{ij}\cdot G_i^2(x)\cdot F_j^2(y)$$

$$\lambda_{ij}=\frac{\left(\frac{\beta_i}{a}\right)^4\int_0^a G_i^{''''}\cdot G_i dx\cdot\int_0^b F_j^2dy+2\cdot\int_0^a\left(\frac{\beta_i}{a}\right)^2 G_i^{''}G_i dx\cdot\int_0^b\left(\frac{\beta_j}{b}\right)^2 F_j^{''}\cdot F_j dy+\int_0^a G_i^2dx\cdot\int_0^b\left(\frac{\beta_j}{b}\right)^4 F_j^{''''}\cdot F_j dy}{\int_0^a G_i^2dx\cdot\int_0^b F_j^2dy}$$

By introducing the notations $\alpha=\frac{b}{a}$, $u=\frac{x}{a}$, $v=\frac{y}{b}$, we obtain the parameter pulsation expression:

$$\lambda_{ij} = \frac{\frac{1}{a^4}[\beta_i^4 \int_0^1 G_i'''' \cdot G_i du \cdot \int_0^1 F_j^2 dv + 2 \cdot \alpha^2 \cdot \beta_i^2 \cdot \beta_j^2 \int_0^1 G_i'' G_i du \cdot \int_0^1 F_j'' \cdot F_j dv + \alpha^4 \cdot \beta_j^4 \int_0^1 G_i^2 du \cdot \int_0^1 F_j'''' \cdot F_j dv}{\int_0^1 G_i^2 du \cdot \int_0^1 F_j^2 dv}$$

For the square plate $\alpha = a/b = 1$ we determine the values of the pulsation parameters, adequate to the normal vibration modes (1,1), (2,1), (3,1).

The values of parameters of the specific pulsations are presented in table

Vibration Mode	Mode (1,1)	Mode (2,1)	Mode (3,1)
$\sqrt{\lambda_{ij}}$, α=1	22,37	61,67	102,9

The modes shapes vibration and the functions $\Phi_{ij}(x, y)$ obtaining using the variation method are:

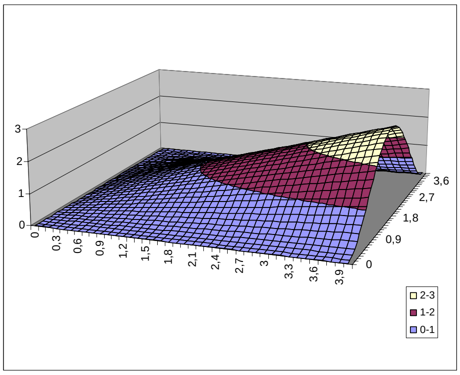

Fig. 1. Mode Shape 11

Table 1

Vibration functions for Mode 11 (i=1, j=1)						
y/x	0	0,4	0,8	1,2	1,6	2
0	0	0	0	0	0	0
0,4	0	0,032754	0,065507	0,098261	0,131015	0,163769
0,8	0	0,107282	0,214564	0,321846	0,429128	0,536409
1,2	0	0,189833	0,379666	0,569498	0,759331	0,949164
1,6	0	0,252091	0,504183	0,756274	1,008366	1,260457
2	0	0,275075	0,55015	0,825226	1,100301	1,375376
2,4	0	0,252092	0,504183	0,756275	1,008367	1,260458
2,8	0	0,189833	0,379666	0,5695	0,759333	0,949166
3,2	0	0,107283	0,214565	0,321848	0,429131	0,536413
3,6	0	0,032755	0,06551	0,098265	0,13102	0,163775
4	0	2,14E-06	4,28E-06	6,43E-06	8,57E-06	1,07E-05
vibration functions for Mode 11 (i=1, j=1)						
y/x	2,4	2,8	3,2	3,6	4	
0	0	0	0	0	0	
0,4	0,196522	0,229276	0,26203	0,294783	0,327537	
0,8	0,643691	0,750973	0,858255	0,965537	1,072819	
1,2	1,138997	1,32883	1,518662	1,708495	1,898328	
1,6	1,512549	1,76464	2,016731	2,268823	2,520914	
2	1,650451	1,925526	2,200601	2,475677	2,750752	
2,4	1,51255	1,764641	2,016733	2,268825	2,520916	
2,8	1,138999	1,328833	1,518666	1,708499	1,898332	
3,2	0,643696	0,750979	0,858261	0,965544	1,072827	
3,6	0,19653	0,229285	0,26204	0,294795	0,32755	
4	1,29E-05	1,5E-05	1,71E-05	1,93E-05	2,14E-05	

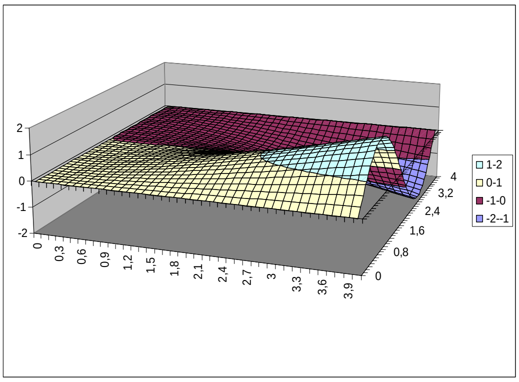

Fig. 2. Mode Shape 21

Table 2

Vibration functions for Mode 21 (i=2, j=1)						
y/x	0	0,4	0,8	1,2	1,6	2
0	0	0	0	0	0	0
0,4	0	0,144501	0,265661	0,343479	0,364099	0,321605
0,8	0	0,473301	0,870148	1,125034	1,192575	1,053389
1,2	0	0,837495	1,539706	1,990721	2,110234	1,863947
1,6	0	1,112165	2,044677	2,643609	2,802318	2,475257
2	0	1,213563	2,231094	2,884633	3,057811	2,700932
2,4	0	1,112165	2,044678	2,643611	2,80232	2,475259
2,8	0	0,837497	1,539709	1,990725	2,110239	1,863951
3,2	0	0,473304	0,870154	1,125042	1,192584	1,053396
3,6	0	0,144507	0,265671	0,343492	0,364114	0,321618
4	0	9,45E-06	1,74E-05	2,25E-05	2,38E-05	2,1E-05
vibration functions for Mode 21 (i=2, j=1)						
y/x	2,4	2,8	3,2	3,6	4	
0	0	0	0	0	0	
0,4	0,218483	0,064626	-0,12508	-0,33436	-0,54999	
0,8	0,715622	0,211675	-0,4097	-1,09518	-1,80143	
1,2	1,266276	0,374555	-0,72495	-1,93789	-3,18759	
1,6	1,681572	0,497396	-0,96271	-2,57345	-4,23302	
2	1,834884	0,542745	-1,05048	-2,80808	-4,61895	
2,4	1,681573	0,497396	-0,96271	-2,57345	-4,23302	
2,8	1,266279	0,374556	-0,72495	-1,93789	-3,1876	
3,2	0,715627	0,211677	-0,4097	-1,09518	-1,80145	
3,6	0,218492	0,064628	-0,12509	-0,33438	-0,55001	
4	1,43E-05	4,23E-06	-8,2E-06	-2,2E-05	-3,6E-05	

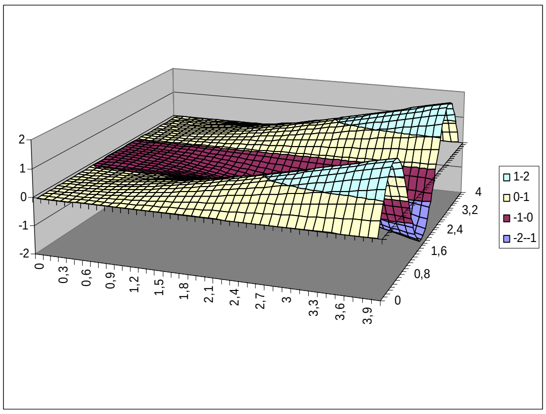

Fig. 3. Mode Shape 31

Table 3

Vibration functions for Mode 31 (i=3, j=1)						
y/x	0	0,4	0,8	1,2	1,6	2
0	0	0	0	0	0	0
0,4	0	0,3482	0,6402	0,8277	0,8774	0,7750
0,8	0	0,9221	1,6952	2,1918	2,3234	2,0522
1,2	0	1,1504	2,1149	2,7345	2,8986	2,5603
1,6	0	0,7905	1,4534	1,8791	1,9919	1,7594
2	0	6E-06	1,1E-05	1,43E-05	1,51E-05	1,34E-05
2,4	0	-0,7905	-1,4533	-1,8791	-1,9919	-1,7594
2,8	0	-1,1503	-2,1144	-2,7344	-2,8986	-2,5603
3,2	0	-0,9220	-1,6951	-2,1917	-2,3233	-2,052
3,6	0	-0,3481	-0,6399	-0,8274	-0,8771	-0,7747
4	0	0,0003	0,0005	0,0007	0,0007	0,0006
vibration functions for Mode 31 (i=3, j=1)						
y/x	2,4	2,8	3,2	3,6	4	
0	0	0	0	0	0	
0,4	0,5265	0,1557	-0,3014	-0,8058	-1,3254	
0,8	1,3942	0,4124	-0,7982	-2,1337	-3,5096	
1,2	1,7394	0,5145	-0,9958	-2,6619	-4,3785	
1,6	1,1953	0,3535	-0,6843	-1,8292	-3,0089	
2	9,08E-06	2,68E-06	-5,2E-06	-1,4E-05	-2,3E-05	
2,4	-1,1952	-0,3535	0,6843	1,8292	3,0088	
2,8	-1,7393	-0,5144	0,9957	2,6618	4,37847	
3,2	-1,3941	-0,4123	0,7981	2,1335	3,5094	
3,6	-0,5263	-0,1556	0,3013	0,8054	1,3249	
4	0,0004	0,0001	-0,0002	-0,0007	-0,0011	

CONCLUSIONS

To validate the suggested method, we compare the achieved theoretical results with the ones determined by other authors when applying the trigonometric series method, Rayleigh, Ritz, Galerkin, Rayleigh-Ritz, the finite difference method and the finite element one. There is a presentation of the percentage deviations of the vibration parameters determined by applying the suggested method in comparison to the ones determined by other authors through the use of the analytical, variation and numerical methods. From the analysis of present data, we note that for the rectangular flat plates considered in the thesis, the percentage deviations are in the limits of high precision.

In literature the results regarding the normal vibration modes for this type of plate are not published, the only references that can be considered are those presented by Leissa, where it is stated that for the case of antisymmetric-antisymmetric vibration mode (2,1), the values of parameters are close to the ones determined in the case of the plate clamped on two opposite edges and free on the other two. Using the Rayleigh method, Janich (1975) determined the value $\sqrt{\lambda_{11}} = 24{,}64$ for the fundamental vibration parameter. The percentage deviation of the fundamental pulsation parameter determined through the suggested method is 9,22 %, in comparison to the value of the pulsation parameter determined through Rayleigh method.

REFERENCES

1. Bors, I., 2007, Aplicatii ale problemei de valori proprii in mecanica constructiilor - sisteme continue. Editura U.T. Pres
2. Fetea, M., 2010, Vibratiile libere ale placilor plane dreptunghiulare cu diverse conditii de contur. Universitatea din Oradea
3. Janich, R., 1975, Die naherungsweise Berechnung der Eigenfrequenzen von rechteckigen Platten bei verschiedenen.
4. Szilard, R., 1974, Theory and Analysis of Plates, Prentice-HallInc., Englewood Cliffs, New Jersey.
5. Marius Fetea., 2008, The aproximate roots for the rectangular plates using the variational Ritz method. Buletinul Universității de Stiințe Agricole şi Medicină Veterinară, vol 65, pag 700, Cluj-Napoca.
6. Marius Fetea., 2008, Theoretical and comparative study regarding the possibility to replace the ferroconcrete plate with „Smart" plate reinforced made by the wood refuse. Buletinul Universității de Stiințe Agricole şi Medicină Veterinară, vol 65(1-2), pag 494, Cluj Napoca.
7. Marius Fetea., 2008, Theoretical and comparative study regarding the mechanics displacements under the static loadings for the plate made by zhe wood refuse and the massif wood. Buletinul Universității de Stiințe Agricole şi Medicină Veterinară, vol 65(1-2), pag 495, Cluj Napoca.
8. Cheregi G., Marius Fetea, 2007, Bending stress and movement from cross section analysis of stacked wood beam, Vol XII, anul 12, Analele universității din Oradea, fascicula Silvicultură
9. Marius Fetea, 2006, Theoretical study concerning at static reply for a circular flat plate loading by distribute uniform forces using the analitycal method. Buletinul Universității de Stiințe Agricole şi Medicină Veterinară, vol 62, pag 207
10. Marius Fetea, 2006: Theoretical and comparative study for establish dynamic reply of a flat square plate having different boundary condition. Buletinul Universității de Stiințe Agricole şi Medicină Veterinară Cluj Napoca, vol 62, pag 208