MECHANICAL DISPLACEMENTS OF WOOD PLATE

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Abstract

The paper presents a variational method analysis for the free vibrations of wood plate. In enginering practice, plate problems often involve consideration of dynamic disturbances, produced by time – dependent external forces or displacements. Dynamic loads may be created by moving vehicles, wind, seismic disturbances, wave impacts, shock or blast, sound, unbalaced machines,etc.

Key words: plate, wood, vibrations modes, variational method.

INTRODUCTION

The rectangular flat plates, as well as the the angular ones in general, often intervene as strength elements in the structures of civil and industrial constructions, their actual shape and support mode being imposed by different conditions in the exploitation of the buildings, such as the lay-out of some technological appliances, pipe crossing, embrasures in the stairs and so on. As for the historic record of the problem related to the study of flat plates, the first results were out for publishing at the end of the 18th century, the beginning of the 19th century, having Chladni E, Strehlke, Konig, R, Tanaka S, Rayleigh L, Ritz W and later on Gontkevich V, Timoshenko S, Leissa as pioneers. Each of the above mentioned authors have had significant contributions regarding the development of methods in order to solve the plates and establish some rigurous solutions of their differential equations of equilibrium. Through a careful analysis of field literature regarding the results obtained for the static and dynamic calculus of flat plates with different boundary conditions, there are to be noted different authors' concerns to elaborate a more exact calculus method that ensures an economical projection in safety conditions.

There has also been noted that most of the complications coming up in solving the flat plates are bound to the existence of free edges, these difficulties being reflected by the impossibility of finding some functions to describe the state of in-plate stress, with the rigurous consideration of the static conditions on a free edge.

MATERIAL AND METHOD

The object of study is determining the vibration modes, self-pulsations ω_{ij} (represented by pulsation parameter λ_{ij}) and the vibration plate functions $W(x, y) = \phi_i(x) \cdot \Psi_j(y)$ .Where, $\Psi_j(y)$ $\phi_i(x)$, are the vibration beam functions for overhang beam There is a unitary presentation of the calculus algorithm of the variational method Galerkin-Vlasov for a plate having clamped one edge and free the others, having differents edge ratios $\alpha = \frac{a}{b} = 1$, $\alpha = \frac{a}{b} = 1{,}5$, $\alpha = \frac{a}{b} = 2$ where a and b, are the lenghts of the edges. In determining these normal vibration modes, the variational method Galerkin-Vlasov is adapted, being regarded as a particularity of the Bubnov-Galerkin method for the dynamic infinite-dimensional systems (Barsan G, Bolotin V). The Bubnov-Galerkin method is applied to reduce partial differential equations governing the dynamics of flexible plates and shells to a discrete system with finite degrees of freedom. The study of the rectangular plate started from the normal vibration mode equation of the plates, that expresses their dynamic equilibrium, choosing for the plate functions the products of the shape functions of the beams with the same boundary conditions as the plate in *x* and respectively *y* direction.

The vibration functions of the beams that have the same boundary conditions as the plate on directions *x* respectively *y* for i,j=1,2,… are (Missir Vlad Ioana, Munteanu Gh, Soare M):

$$\phi_1(x) = 1$$

$$\phi_2(x) = \sqrt{3} \cdot (1 - \frac{2 \cdot x}{a})$$

....

$$\phi_i(x) = \left(\cosh \beta_i \frac{x}{a} + \cos \beta_i \frac{x}{a}\right) - k_i \cdot \left(\sinh \beta_i \frac{x}{a} + \sin \beta_i \frac{x}{a}\right)$$

$$\Psi_j(x) = \left(\cosh \beta_j \frac{y}{b} - \cos \beta_j \frac{y}{b}\right) - k_j \cdot \left(\sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b}\right).$$

Where, $\beta_i, \beta_j, k_i, k_j$, represents parameters having the following values:

Table 1

Parameters of the beam on y direction

Nr	β_j	k_j
1	1,875104	0,734096
2	4,614091	1,018466
3	7,854757	0,999225

Table 2

Parameters of the beam on x direction

Nr	β_i	k_i
1	4,73004	0,982502
2	7,853204	1,000777
3	10,995607	0,999966

The equation of the normal vibration modes

$$\nabla^4 W_{ij}(x,y) = \lambda_{ij} \cdot W_{ij}(x,y),$$

in which ∇^4 is the double Laplacean operator, together with the boundary conditions, represents a Sturm-Liouville problem, whose solving with the suggested method leads to the charateristics of pulsations and vibration shapes (Ciofoaia V, Botis M).

The expression of the parameter of specific pulsations obtained by applying the suggested method is

$$\lambda_{ij} = \frac{\frac{1}{a^4}[\beta_i^4 \int_0^1 \phi_i^{''''} \cdot \phi_i du \cdot \int_0^1 \Psi_j^2 dv + 2 \cdot \alpha^2 \cdot \beta_i^2 \cdot \beta_j^2 \int_0^1 \phi_i^{''} \phi_i du \cdot \int_0^1 \Psi_j^{''} \cdot \Psi_j dv + \alpha^4 \cdot \beta_j^4 \int_0^1 \phi_i^2 du \cdot \int_0^1 \Psi_j^{''''} \cdot \Psi_j dv}{\int_0^1 \phi_i^2 du \cdot \int_0^1 \Psi_j^2 dv}$$

The use of Galerkin-Vlasov method for determining the normal vibration modes of the plates is reduced to the evaluation of the integrals defined above.

Table 3

The integrals for the beam on y direction

Nr. Crt	Integrala	Valoarea
1	$\int_0^1 \Psi_1^2 du$	1
2	$\int_0^1 \Psi_2^2 du$	1
3	$\int_0^1 \Psi_3^2 du$	1
4	$\int_0^1 \Psi_1 \cdot \Psi_1^{''} du$	0,858244
5	$\int_0^1 \Psi_2 \cdot \Psi_2^{''} du$	-12,882326
6	$\int_0^1 \Psi_3 \cdot \Psi_3^{''} du$	-45,904227

7	$\int_0^1 \Psi_1 \cdot \Psi_1'''' du$	12,362363
8	$\int_0^1 \Psi_2 \cdot \Psi_2'''' du$	485,518819
9	$\int_0^1 \Psi_3 \cdot \Psi_3'''' du$	3806,546266

Table 4

The integrals for the beam on x direction

Nr. Crt	Integrala	Valoarea
1	$\int_0^1 \phi_1^2 du$	1
2	$\int_0^1 \phi_2^2 du$	1
3	$\int_0^1 \phi_3^2 du$	1
4	$\int_0^1 \phi_1 \cdot \phi_1'' du$	0
5	$\int_0^1 \phi_2 \cdot \phi_2'' du$	0
6	$\int_0^1 \phi_3 \cdot \phi_3'' du$	-12,302620
7	$\int_0^1 \phi_1 \cdot \phi_1'''' du$	0
8	$\int_0^1 \phi_2 \cdot \phi_2'''' du$	0
9	$\int_0^1 \phi_3 \cdot \phi_3'''' du$	500,563890

RESULTS AND DISSCUSIONS

For the studied plate, the pulsation parameters for a maximum number of 3 normal vibration modes are determined. The obtained results (Fetea M. et. al, 2009) regarding the pulsation parameters for the studied flat rectangular plates are subsequently presented

- for $\alpha = 1$, λ_{11}=12,36; λ_{21}=12,36; λ_{31}=385,93
- for $\alpha = 1{,}5$, λ_{11}=27,81; λ_{21}=27,81; λ_{31}=384,04

- for $\alpha = 2$, λ_{11}=49,44; λ_{21}=49,44; λ_{31}=382,42

Having the parameters of pulsations, using the expression (Timoshenko St)

$$\omega_{ij} = \sqrt{\frac{\lambda_{ij} \cdot D}{\rho}} = \frac{1}{a^2} \cdot \sqrt{\frac{\lambda_{ij} \cdot D}{\rho}},$$

it can be determinate the self pulsations of plates for each mode (Fetea M. et. al, 2009), table 5.

Table 5

Self -pulsations

Plate	ω11	ω21	ω31
$\alpha = 1$	0,2198	0,2198	1,4443
$\alpha = 1{,}5$	0,4944	0,4944	1,5445
$\alpha = 2$	0,6215	0,6215	1,6345

The vibration functions (Fetea M. et. al, 2009) of plate are presented in table 6 and the deformation of plate in figure 1, 2 and 3.

Table 6

The functions vibrations of plate for mode i=1, j=1

y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
0,4	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
0,8	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
1,2	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
1,6	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
2	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
2,4	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
2,8	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,922	1,1817	1,4509	1,7247	1,9999
3,2	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
3,6	0	0,0335	0,1277	0,2729	0,4597	0,6790	0,9222	1,1817	1,4509	1,7247	1,9999
4	0	0,0335	0,1277	0,2729	0,4597	0,679	0,9222	1,1817	1,4509	1,7247	1,9999
The functions vibrations of plate for mode i=2, j=1											
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,901	3,4495	3,9999
0,4	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,901	3,4495	3,9999
0,8	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,901	3,4495	3,9999
1,2	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,901	3,4495	3,9999
1,6	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
2	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
2,4	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
2,8	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
3,2	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
3,6	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
4	0	0,0670	0,2554	0,5459	0,9195	1,3580	1,8445	2,363	2,9019	3,4495	3,9999
The functions vibrations of plate for mode i=3, j=1											
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0,0670	0,2554	0,545	0,919	1,358	1,844	2,363	2,9019	3,449	3,999
0,4	0	0,0360	0,1372	0,293	0,493	0,729	0,990	1,269	1,5588	1,853	2,1486
0,8	0	0,0065	0,0249	0,053	0,089	0,132	0,180	0,2308	0,2835	0,337	0,3909
1,2	0	-0,0182	-0,069	-0,148	-0,250	-0,369	-0,501	-0,642	-0,7893	-0,938	-1,088

1,6	0	-0,0349	-0,1329	-0,284	-0,478	-0,706	-0,959	-1,229	-1,509	-1,794	-2,080
2	0	-0,0407	-0,1552	-0,331	-0,558	-0,825	-1,121	-1,436	-1,763	-2,096	-2,431
2,4	0	-0,0349	-0,1329	-0,284	-0,478	-0,706	-0,959	-1,229	-1,509	-1,794	-2,080
2,8	0	-0,0182	-0,0694	-0,148	-0,250	-0,369	-0,501	-0,642	-0,789	-0,938	-1,088
3,2	0	0,0065	0,0249	0,053	0,089	0,1327	0,1801	0,2309	0,2836	0,337	0,3909
3,6	0	0,0360	0,1372	0,293	0,493	0,7295	0,9908	1,2695	1,558	1,853	2,148
4	0	0,0670	0,2554	0,545	0,919	1,3581	1,844	2,363	2,901	3,449	4,000

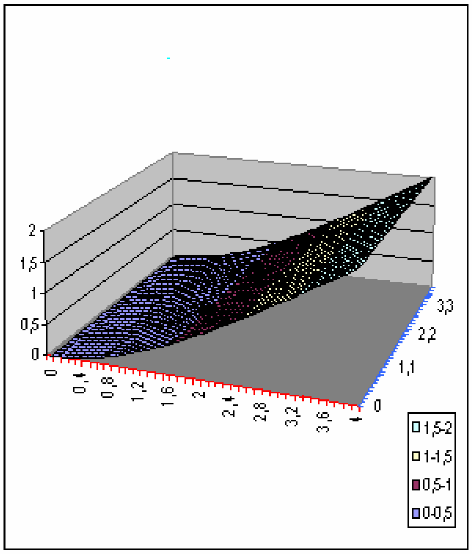

Fig.1 The plate deformation for mode i=1, j=1

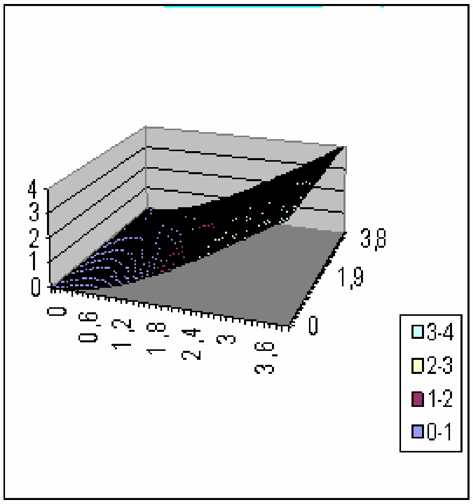

Fig.2 The plate deformation for mode i=2, j=1

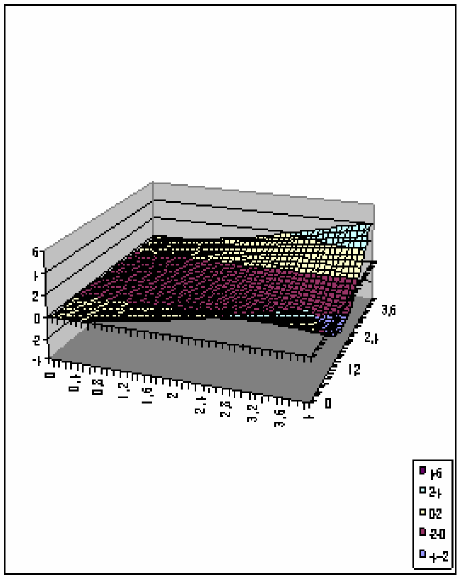

Fig.3 The plate deformation for mode i=3, j=1

CONCLUSIONS

The results of the scientific research regarding the flat plates, as well as the practical importance related to the knowledge of their way of behaviour in different loading and support situations within some structures (machines, buildings, equipments) are emphasized in numerous treaties, books, scientific papers published throughout the centuries.

The dynamic analysis of plates there has been an emphasis on the complexity of the notion of dynamic calculus, which has the following as main working stages (Forray M):

- establishing the dynamic model considered for the first time;
- determining the normal vibration modes (self pulsations and vectors, respectively the functions of vibration modes);
- determining the dynamic response in displacements and sectional stresses;
- checking stability and strength conditions.

Through a careful analysis of field literature regarding the results obtained for the static and dynamic calculus of flat plates with different boundary conditions, there are to be noted different authors' concerns to elaborate a more exact calculus m ethod that ensures an economical projection in safety conditions.

There has also been noted that most of the complications coming up in solving the flat plates are bound to the existence of free edges, these difficulties being reflected by the impossibility of finding some functions to describe the state of in-plate stress, with the rigurous consideration of the static conditions on a free edge. In this paper

From the study of field literature it has been noted that there is no data concerning the results obtained by other authors regarding the values of the shape functions, respectively the parameters of the rectangular plate pulsations and the pulsations proper by means of applying the Galerkin-Vlasov variational method, a reason why the results presented in the paper represent a novelty element brought by the author.

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