FREE VIBRATION ANALYSIS OF A SIMPLY SUPPORTED RECTANGULAR WOOD PLATE USING A VARIATIONAL METHOD

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Abstract

The undamped free flexural vibrations of rectangular plates are basically boudary value problems of the mathematical physics. Since the solution in the case of freely vibrating plates reduces to the honogeneous differential equations of motion, the variational method described in the paper for the solution of the homogeneousbiharmonic equations can be extended. So, in this paper was investigated the solution of differential equation of undamped motion.using yhe shape function of the beams simply supported to the limits by the X and Y directions.

Key words: plate, wood, variational, free, vibrations.

INTRODUCTION

Initially, in the case of rectangular plate it was considered that is sufficient be taken into account only the resonance phenomenon. Later it being understood that vibration systems located away from the resonance can even modify and alter the structure of the materials. The literature (Bârsan G, 1979), (Ille V, et al, 1981), (Szilard R, 1974), (Szilard R. et al, 1965), reflected that the assessment respectively static and dynamic response of flat plate is done taking into account the shape of the median surface, boundary conditions, loading mode and solving method adopted. In this paper was adopted the Galerkin-Vlasov variational method for solving the free vibrations problem.

MATERIAL AND METHOD

We initially adopted the following assumptions for calculating: flat plate analyzed is considered as being thin, elastic, isotropic with bending stiffness and meets the conditions of validity of Kirchhoff's hypothesis (Claassen R, et al, 2000), (Fetea M., 2009), (Hamada, M 1959), (Timoshenko P, 1962).

The proposed calculation method is an adaptation of variational method Galerkin-Vlasov (Fetea M., 2009), (Szilard R,1974), (Szilard R et al, 1965) for static flat plates and was so elaborate that the calculations necessary to determine the characteristics of the plate are made on the basis of programs prepared by the author (Fetea M, 2009). Is considered a rectangular plate presented in figure 1, with dimensions a and b simply supported in the sides $x = 0, x = a$ and simply supported on the side $y = 0$, $y = b$.

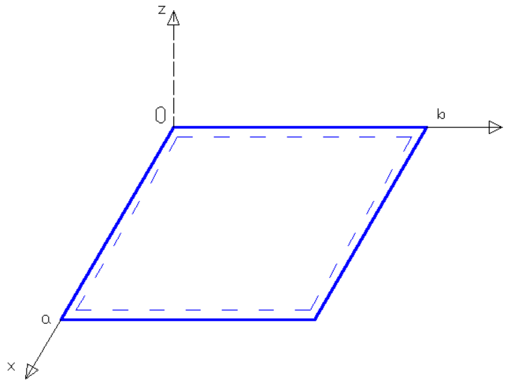

Fig. 1 The median surface of plate

On the median surface is considered in figure 2 a network in which nodes are determined the natural vibration shapes function values (Fetea M., 2009).

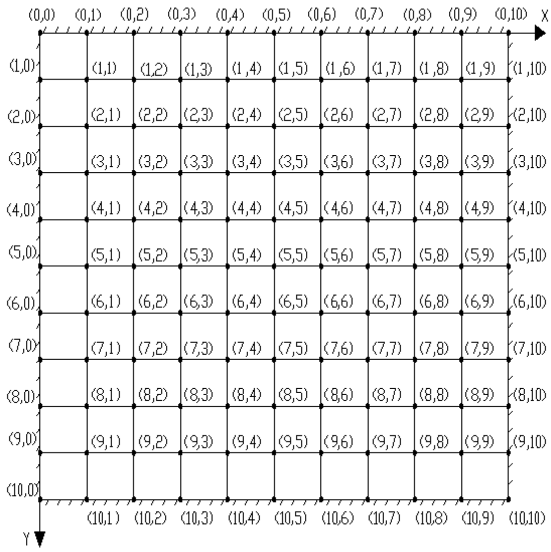

Fig. 2 The median surface network

The shape functions of beams for the first three modes of free vibrations are presented in figure 3 (Fetea M., 2009).

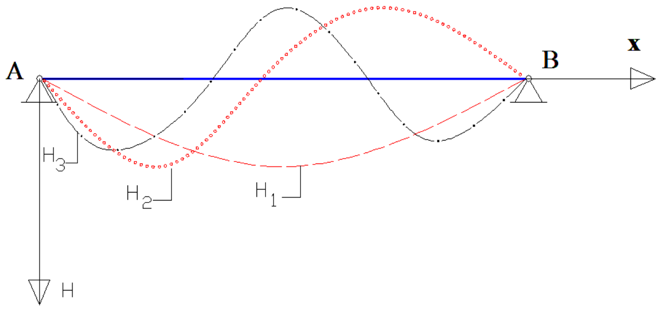

Fig. 3 The shape functions of simply supported beam

The expressions of beams shapes functions of vibration on the X and Y directions are (Borş I, 2005):

$$H_i(x) = \sin \beta_i \frac{x}{a},$$

$$H_j(y) = \sin \beta_j \frac{y}{b}.$$

The parameter values $\beta_i, k_i,\ \beta_j, k_j$, were determined (Borş I, 2005): The expressions of shapes functions (Bârsan G, 1979), (Bârsan G, 1971) (Janich R, 1962), (Roşca I, 2002) for the plate are:

$$\Phi_{ij}(x, y) = H_i(x) \cdot H_j(y)$$

Sturm-Liouville problem (Borş I, 2005) associated with flat square plate is:

$$\nabla^4 \Phi_{ij}(x, y) = \lambda_{ij} \Phi_{ij}(x, y)$$

$$\Phi_{ij}(0, y) = 0, \frac{\partial^2 \Phi_{ij}}{\partial x^2}(0, y) = 0,$$

$$\Phi_{ij}(a, y) = 0, \frac{\partial^2 \Phi_{ij}}{\partial x^2}(a, y) = 0,$$

$$\Phi_{ij}(x, 0) = 0, \frac{\partial^2 \Phi_{ij}}{\partial y^2}(x, 0) = 0,$$

$$\Phi_{ij}(x, b) = 0, \frac{\partial^2 \Phi_{ij}}{\partial y^2}(x, b) = 0.$$

Where, λ_{ij} represent the pulsations parameters for i anj modes of free vibration by X and Y directions of simply supported plate. Substituting equation functions of normal modes through their functions of beams products, integrating relationships across the plate and using the method of separation of variables is obtain (Filimon, I et al, 1983), (Soare M, 1999), (Leissa A, 1969):

$$\left(\frac{\beta_i}{a}\right)^4 \int_0^a (H_i^{IV}(x)dx \cdot \int_0^b H_j^2(y)dy + 2\left(\frac{\beta_i}{a}\right)^2 \left(\frac{\beta_j}{b}\right)^2 \int_0^a H_i''(x) \cdot H_i(x)dx \cdot$$

$$\int_0^b H_j''(y) \cdot H_j(y)dy + \left(\frac{\beta_j}{b}\right)^4 \int_0^a H_i^2(x)dx \cdot \int_0^b H_j^{IV}(y)H_j(y) = \lambda_{ij} \cdot H_i^2(x) \cdot H_j^2(y),$$

The expression of the pulsations parameters is (Fetea M., 2009):

$$\lambda_{ij} = \frac{\left[\begin{aligned}&\left(\frac{\beta_i}{a}\right)^4 \cdot \int_0^a H_i^{IV}(x)dx \cdot \int_0^b H_j^2(y)dy + 2 \cdot \left(\frac{\beta_i}{a}\right)^2 \cdot \left(\frac{\beta_j}{b}\right)^2 \int_0^a H_i''(x) \cdot H_i(x)dx \cdot \int_0^b H_j''(y) \cdot H_j(y)dy + \\ &+ \left(\frac{\beta_j}{b}\right)^4 \cdot \int_0^a H_i^2(x)dx \cdot \int_0^b H_j^{IV}(y)H_j(y)\end{aligned}\right]}{H_i^2(x) \cdot H_j^2(y)}$$

We adopt the notations (Fetea M., 2009), (Szilard R, 1974):

$$I_1 = \left(\frac{\beta_i}{a}\right)^4 \cdot \int_0^1 H_i^{IV}(x) \cdot H_i(x)dx = \left(\frac{\beta_i}{a}\right)^4 \int_0^a \left[\left(\sin \beta_i \frac{x}{a}\right)^{IV} \left(\sin \beta_i \frac{x}{a}\right)\right]dx$$

$$I_2 = \int_0^a H_i^2(x)dx = \int_0^a \sin^2\left(\beta_i \frac{x}{a}\right)dx,\ I_3 = \int_0^b H_j^2(y)dy = \int_0^b \sin^2\left(\beta_j \frac{y}{b}\right)dy,$$

$$I_4 = \left(\frac{\beta_i}{a}\right)^2 \int_0^a H_i''(x) \cdot H_i(x)dx = \left(\frac{\beta_i}{a}\right)^2 \int_0^a \left(\sin \beta_i \frac{x}{a}\right)'' \cdot \left(\sin \beta_i \frac{x}{a}\right)dx$$

$$I_5 = \left(\frac{\beta_j}{b}\right)^2 \int_0^a H_j''(y) \cdot H_j(y)dy = \left(\frac{\beta_j}{b}\right)^2 \int_0^b \left(\sin \beta_j \frac{y}{b}\right)'' \cdot \left(\sin \beta_j \frac{y}{b}\right)dy,$$

$$I_6 = \left(\frac{\beta_j}{b}\right)^4 \int_0^b H_j^{IV}(y) \cdot H_j(y)dy = \left(\frac{\beta_j}{b}\right)^4 \int_0^b \left[\left(\sin \beta_j \frac{y}{b}\right)^{IV} \cdot \left(\sin \beta_j \frac{y}{b}\right)\right]dy$$

By entering the integrals in the parameter expression are obtained their values.

RESULTS AND DISSCUSIONS

For the simply supported plate having different ratio of sides the pulsations parameters values for normal modes of vibration (1,1), (2,1), (3,1) is shown in table 1. The values of the natural vibration shapes functions are shown in tables 2-4 and their shapes corresponding to the normal modes of free vibrations in figures 4-6.

Table 1

Pulsations parameters

Vibrations modes	Mode (1,1)	Mode (2,1)	Mode (3,1)
$\sqrt{\lambda_{ij}}\ ,\alpha=1$	19.7392	49,34	78,65
$\sqrt{\lambda_{ij}}\ ,\alpha=1{,}5$	32,07	61,68	111.03
$\sqrt{\lambda_{ij}}\ ,\alpha=2$	49,34	78,95	128,3

Table 2

Shapes functions for mode (1,1)

		Vibrations functions							Mode (1,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,0954	0,1814	0,25	0,2938	0,3090		0,25	0,1816	0,0954	-1,4E-08
0,8	0	0,1816	0,3454	0,4755	0,5590	0,5877	0,5590	0,4755	0,3454	0,1816	-2,7E-08
1,2	0	0,25	0,4755	0,6545	0,7694	0,8090	0,7694	0,6545	0,4755	0,25	-3,8E-08
1,6	0	0,2938	0,5590	0,7694	0,9045	0,9510	0,9045	0,7694	0,559	0,2938	-4,4E-08
2	0	0,3090	0,5877	0,8090	0,9510	1	0,9510	0,8090	0,587785	0,3090	-4,6E-08
2,4	0	0,2938	0,5590	0,7694	0,9045	0,9510	0,9045	0,7694	0,559017	0,2938	-4,4E-08
2,8	0	0,25	0,4755	0,6545	0,7694	0,8090	0,7694	0,6545	0,475528	0,25	-3,8E-08
3,2	0	0,1816	0,3454	0,4755	0,5590	0,5877	0,5590	0,4755	0,345491	0,1816	-2,7E-08
3,6	0	0,0955	0,1816	0,25	0,2938	0,3090	0,2938	0,25	0,181636	0,0954	-1,4E-08
4	0	-1,4E-08	-2,7E-08	-3,8E-08	-4,4E-08	-4,6E-08	-4,4E-08	-3,8E-08	-2,7E-08	-1,4E-08	2,15E-15

Table 3

Shapes functions for mode (2,1)

		Vibrations functions							Mode (2,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,1816	0,3454	0,4755	0,5590	0,5877	0,5590	0,4755	0,3454	0,1816	-2,7E-08
0,8	0	0,2938	0,5590	0,7694	0,9045	0,9510	0,9045	0,7694	0,5590	0,2938	-4,4E-08
1,2	0	0,2938	0,5590	0,7694	0,9045	0,95105	0,9045	0,7694	0,5590	0,2938	-4,4E-08
1,6	0	0,1816	0,3454	0,4755	0,5590	0,5877	0,5590	0,4755	0,3454	0,1816	-2,7E-08
2	0	-1,4E-08	-2,7E-08	-3,8E-08	-4,4E-08	-4,6E-08	-4,4E-08	-3,8E-08	-2,7E-08	-1,4E-08	2,15E-15
2,4	0	-0,1816	-0,3454	-0,4755	-0,5590	-0,5877	-0,5590	-0,4755	-0,3454	-0,1816	2,73E-08
2,8	0	-0,2938	-0,5590	-0,7694	-0,9045	-0,9510	-0,9045	-0,7694	-0,5590	-0,2938	4,41E-08
3,2	0	-0,2938	-0,5590	-0,7694	-0,9045	-0,9510	-0,9045	-0,7694	-0,5590	-0,2938	4,41E-08
3,6	0	-0,1816	-0,3454	-0,4755	-0,5590	-0,5877	-0,5590	-0,4755	-0,3454	-0,1816	2,73E-08
4	0	2,87E-08	5,46E-08	7,51E-08	8,83E-08	9,28E-08	8,83E-08	7,51E-08	5,46E-08	2,87E-08	-4,3E-15

Table 4

Shapes functions for mode (2,1)

		Vibrations functions							Mode (3,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,25	0,4755	0,65450	0,7694	0,8090	0,7694	0,6545	0,4755	0,25	-3,8E-08
0,8	0	0,2938	0,5590	0,7694	0,9045	0,9510	0,9045	0,7694	0,5590	0,2938	-4,4E-08
1,2	0	0,0954	0,1816	0,25	0,2938	0,3090	0,2938	0,25	0,1816	0,0954	-1,4E-08
1,6	0	-0,1816	-0,3454	-0,4755	-0,559	-0,5877	-0,5590	-0,4755	-0,3454	-0,1816	2,73E-08
2	0	-0,3090	-0,5877	-0,8090	-0,9510	-1	-0,9510	-0,8090	-0,5877	-0,3090	4,64E-08
2,4	0	-0,1816	-0,3454	-0,4755	-0,5590	-0,5877	-0,5590	-0,4755	-0,3454	-0,1816	2,73E-08
2,8	0	0,0954	0,1816	0,25	0,2938	0,3090	0,2938	0,25	0,1816	0,0954	-1,4E-08
3,2	0	0,2938	0,5590	0,7694	0,9045	0,9510	0,9045	0,76942	0,5590	0,2938	-4,4E-08
3,6	0	0,25	0,4755	0,6545	0,7694	0,8090	0,769	0,6545	0,4755	0,25	-3,8E-08
4	0	-4,3E-08	-8,2E-08	-1,1E-07	-1,3E-07	-1,4E-07	-1,3E-07	-1,1E-07	-8,2E-08	-4,3E-08	6,46E-15

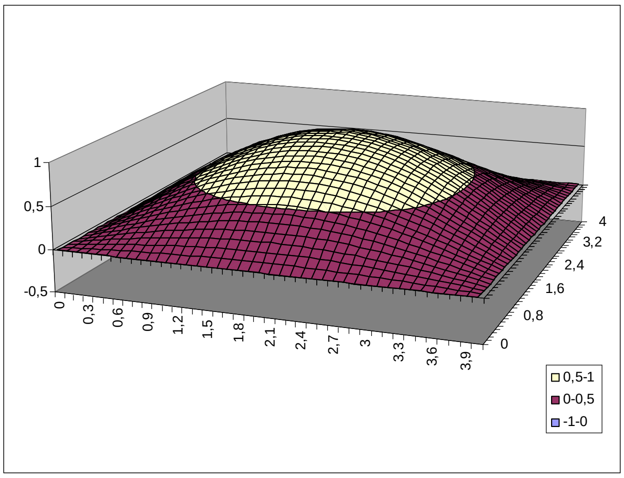

Fig. 4 The Image of shape corresponding to mode (1,1)

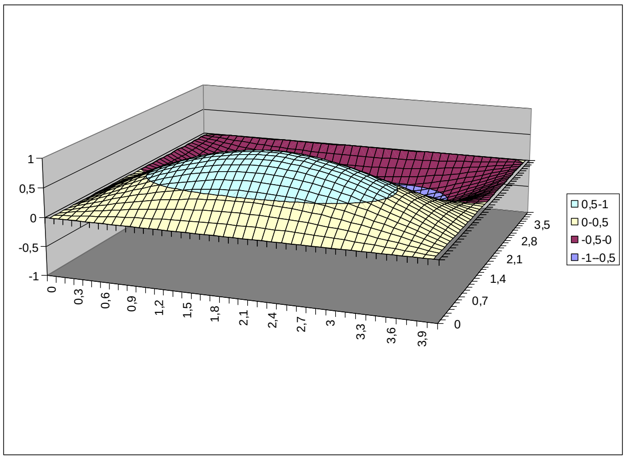

Fig. 5 The Image of shape corresponding to mode (2,1)

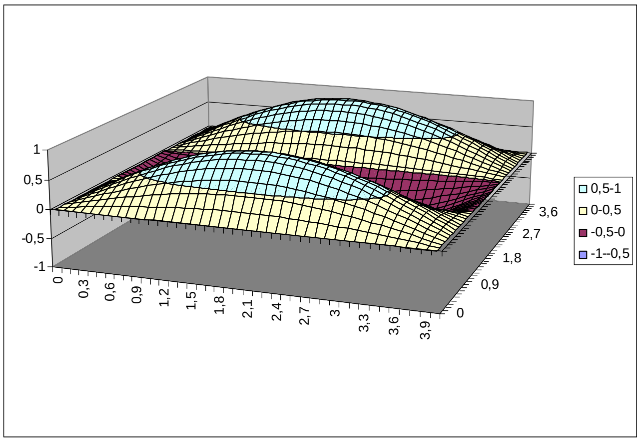

Fig. 6 The Image of shape corresponding to mode (3,1)

CONCLUSIONS

In the case of Galerkin-Vlasov variational method adapted by the author are highlighted as follows (Fetea M., 2009):

- The election of displacement function approximation as a linear combination between the natural vibration shapes products functions of the beams on both directions and time function which indicates that plate motion after a normal harmonic vibration which is a motion that is produce a specific pulsation;
- Calculation algorithm, which characterizes the method;
- Pulsations parameters considered for normal modes of vibration;
- The shapes funcions for three normal modes of vibration.

In table 5 is presents the parameter values pulsations for the square plate and the percentage deviations of the values obtained by the proposed method to those determined by other authors.

As a conclusion of this study the Galerkin -Vlasov method uses linear combination of the eigenfunctions of lateral beams vibrations, which are able to satisfy most boundary conditions.

Table 5

Parameters pulsations for the square plate

Parametrii pulsațiilor	$\sqrt{\lambda_{11}}$	$\sqrt{\lambda_{21}}$	$\sqrt{\lambda_{31}}$
The proposed method	19.7392	49,34	78,65
Polidor Bratu [74] Levy Method	19,74 0,06 %	49,4 0,13 %	79,05 0,51 %
Szilard[82] Rayleigh Method	19,722 0,08 %	50,6 2,5 %	-
Szilard[82] Finite elements method	19,73 0 %	49,34 0 %	78,95 0,31 %
Bârsan [4]	19,739 0 %	49,350 0,03 %	78,973 0,41 %

REFERENCES

1. Bârsan, G., 1979, Dinamica şi stabilitatea construcţiilor, Editura Didactică şi Pedagogică Bucureşti, pp 45-76
2. Bârsan, G., M., 1971 Vibraţiile şi stabilitatea plăcilor poligonale. In Ph.D. Thesis Cluj-Napoca.
3. Borş, I., 2005, Aplicaţii ale problemei de valori proprii în mecanica construcţiilor – Sisteme finit-dimensionale, Editura U.T. Pres, Cluj-Napoca.
4. Claassen, R., W., Thorne, C., 2000 Transverse Vibrations of Thin Rectangular Isotropic Plates, Nots. Tech. Pub. 2379, NAVWEPS Rept. 7016 U.S. Naval Ordnance Test Sta., China Lake, Calif., Aug. 1960.Dieter, G.: Engineering Design. Boston. McGraw Hill.
5. Fetea, M., 2009, Vibratiile placilor plane cu diverse conditii de contur In: Ph.D. Thesis, *Technical* University of Cluj-Napoca, Cluj-Napoca, Romania.
6. Filimon, I., Soare, M 1983., Ecuaţii diferenţiale cu aplicaţii în mecanica construcţiilor, Editura Tehnică, Bucureşti.
7. Fletcher, H., J: The Frequency of Vibration of Rectangular Isotropic Plates, J. Appl. Mech., vol. 26, no. 2, June, 1959, p.290.
8. Hamada, M 1959., Method of Solving Problems of Vibration, Deflection and Buckling of Rectangular Plates With Clamped or Supported Edges, Bull JSME, vol. 2, no. 5, 1959, pp. 92-97.
9. Hamada, M., 1959, Compressive or Shearing Buckling Load and Fundamental Frequency of a Rhomboidal Plate With All Edges Clamped. Bull. JSME, vol. 2, no. 8, Nov., pp. 520-526.
10. Iguchi, S: Die Eigenwertprobleme fur die elastische rechteckige Platte, Mem. Fac. Eng., Hokkaido Univ., 1938, pp. 305-372.
11. Ille, V., Bia, C., Borş, I.,1981 Plăci plane dreptunghiulare având una, două sau trei laturi libere, Sesiunea ştiinţifică a Institutului Politehnic Iaşi, pag. 190-193.
12. Janich, R 1962., Die naherungsweise Berechnung der Eigenfrequenzen von rechteckigen Platten bei verschiedenen Randbedingungen, Die Bautechnik, vol. 3, Mar. 1962, pp. 93-95.
13. Leissa, A., W., 1969 Vibration of plates, NASA SP. 160, Washington, pp 26-48.
14. Roşca, I., C., 2002 Vibratiile sistemelor elastice, Editura Infomarket,
15. Soare, M., 1999., Ecuaţii diferenţiale cu aplicaţii în mecanica construcţiilor, Editura Tehnică, Bucureşti,.
16. Szilard, R., Hubka W. F., 1965 Static and Dynamic Analysis of Forced Vibrations of Arbitrary Shape and Boundary Condition, Publ. Intern. Assoc. Bridge Struct. Eng., 25, 317-338.
17. Szilard, R.,1974, Theory and Analysis of Plates, Prentice-HallInc.,Englewood Cliffs, New Jersey
18. Timoshenko, St., P., 1962 Strenght of Materials, Part I Elementary Theory and Problems, 3-rd Edition, Part II Advaced Theory and Problems, 3-rd Edition1962, D. Van Nostrand Company, Inc., Princeton, New Jersey.