STUDIES REGARDING MODAL DISPLACEMENTS AND AXIAL SECTIONAL EFORTS AT WOOD PROCESSING CNC SPINDLE MACHINE TOOLS

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Abstract

Dynamics of woodworking machine tools CNC spindle, represents currently a problem waiting to be solved entirely.The free and forced oscillations that occur into the structure during processing, represents dangerous phenomenom in woodworking process. Knowing the normal modes of vibration, consttituie one starting point to determine the amortization methods for reducing the vibrations values.

Key words: spindle, modal, efforts, numerical, method.

INTRODUCTION

In the paper the problem of modal vibrations analysis and axial efforts calculus was done for the principal spindle of the drilling operations at the CNC machine tool. It was considered as a machine tool CNC processing by drilling the wood material. The studies conducted by numerical methods are materialized by the results that have been obtained on modal and total displacements for free and forced vibrations, as well as the axial sectional efforts values. Study and their determination was made by creating programs using Matlab software.

MATERIAL AND METHOD

For free longitudinal vibrations, the modal displacements expression is (William M., 2005; Bratu P., 2000):

$$u_i(x,t) = \sin\left(\frac{p_i}{c}x\right)\cdot\cos(p_i t - \varphi_i) \qquad (1)$$

Where:

$u_i(x,t)-$ the longitudinal modal displacements;

$p_i -$ spindle pulsation;

$i -$ modes number of vibrations;

$\varphi_i -$ initial phase of harmonic motion;

$c -$ elastic wave propagation speed;

E – Young's modulus;
ρ – density of material (OLC45);
l – spindle length.

$$c^2 = \frac{E}{\rho} \tag{2}$$

$$p_i = (2i-1)\frac{\pi}{2l}\sqrt{\frac{E}{\rho}} \quad \text{(Ispas C., 1986),} \tag{3}$$

It adopts the hypothesis that the initial phase of the movement is null (Pop C. et al., 1999; Fetea M., 2010; Albu A., 1995).

The total displacements shall be determined applying the principle of superposition effects, by the relationship (Bratu P., 2000):

$$u(x,t) = \sum_{i=1}^{\infty} u_i(x,t) = \sum_{i=1}^{\infty} \sin\left(\frac{p_i}{c}x\right)\cos(p_i t) \tag{4}$$

At the considered moments:

$$t_1 = 0.1[\text{sec}],$$
$$t_2 = 0.2[\text{sec}],$$
$$t_3 = 0.3[\text{sec}],$$

were determined the modal and total displacements values for free from vibrations of the CNC main spindle.

The following calculation was performed using Matlab software program (Morar L., 2006):

```
E=2.1*(10^5) % Young's modulus
d=30 % spindle diameter [mm]
L=30 % length spindle [mm]
A=7.85 % spindle material density
pF=300 % disturbing force pulsation
F0=136.0123 % modulus of harmonic disturbing forces
t1=.1
t2=.5
t3=1
F01=F0*cos(pF*t1)
F02=F0*cos(pF*t2)
F03=F0*cos(pF*t3)
Aria=(pi*d^2)/4 % area
c=sqrt(E/A)
x=0:50:300
for i=1
ppr1=(2*i-1)*((pi*c)/(2*L)) % own pusation mode 1
end
for j=2
ppr2=(2*j-1)*((pi*c)/(2*L)) % own pusation mode 2
end
for k=3
```

```
ppr3=(2*k-1)*((pi*c)/(2*L)) % own pusation mode 3
       end
       % Shapes functions values
       for i=1
Fpr1=sin((2.*i-1)*((pi)/2*L).*x)% shape function mode 1
       end
       for j=2
Fpr2=sin((2.*j-1)*((pi)/2*L).*x)% shape function mode 2
end
       for k=3
Fpr3=sin((2.*k-1)*((pi)/2*L).*x)% shape function mode 3
       end
       % Displacement corresponding to mode 1
for t1=1
u1=(sin((ppr1/c)*x))*cos(ppr1*t1)
       end
       % Displacement corresponding to mode 2
       for t2=2
       u2=(sin((ppr2/c)*x))*cos(ppr2*t2)
       end
       % Displacement corresponding to mode 3
       for t3=3
       u3=(sin((ppr3/c)*x))*cos(ppr3*t3)
       end
       % Total axial displacement
       U=u1+u2+u3
```

Having the modal and total displacements values obtained for spindle free vibrations, can be determinate modal displacements by axial cutting forces load components, using the shapes functions and the dynamic multiplicator.expressions (Albu A., 1995; Fetea M., 2010).

$$X_i(x) = \frac{(2i-1)\pi}{2l}x$$
$$u_i(x,t) = X_i(x)\cdot\Psi_i(t) = \frac{(2i-1)\pi}{lEA}\frac{1}{1-\left(\frac{\omega}{p_i}\right)^2}\sin\omega t_i \tag{5}$$

Where:

$\Psi_i(t) -$ dynamic multiplicators;

$X_i(x) -$ shapes function by X zxis;

$\omega -$ disturbing forces pulsation.

Dynamic displacements of forced oscillations is determined using the superposition principle (Bratu P., 2000):

$$u(x,t) = \sum_{i=1}^{\infty} u_i(x,t) \tag{6}$$

The following calculation was performed using Matlab software program (Morar L., 2006):

```
E=2.1*(10^5) % Young's modulus
d=30 % spindle diameter [mm]
L=30 % length spindle [mm]
A=7.85 % spindle material density
pF=300 % disturbing force pulsation
F0=136.0123 % modulus of harmonic disturbing forces
t1=.1
t2=.5
t3=1
F01=F0*cos(pF*t1)
F02=F0*cos(pF*t2)
F03=F0*cos(pF*t3)
Aria=(pi*d^2)/4
c=sqrt(E/A)
x=0:50:300
for i=1
   ppr1=(2*i-1)*((pi*c)/(2*L)) % spindle pulsation mode 1
end
for j=2
    ppr2=(2*j-1)*((pi*c)/(2*L)) % spindle pulsation mode 2
end
for k=3
    ppr3=(2*k-1)*((pi*c)/(2*L)) % spindle pulsation mode 3
end
% Main Spindle Shapes functions
for i=1
   Fpr1=sin((2.*i-1)*((pi)/2*L).*x) % mode 1
end
for j=2
   Fpr2=sin((2.*j-1)*((pi)/2*L).*x)% mode 2
end
for k=3
   Fpr3=sin((2.*k-1)*((pi)/2*L).*x)% mode 3
end
% Displacement corresponding to mode 1
for t1=1
   u1=(sin((ppr1/c)*x))*cos(ppr1*t1)
end
% Displacement corresponding to mode 2
for t2=2
   u2=(sin((ppr2/c)*x))*cos(ppr2*t2)
end
% Displacement corresponding to mode 3
for t3=3
  u3=(sin((ppr3/c)*x))*cos(ppr3*t3)
end
% Axial total displacements
```

```
U=u1+u2+u3
% Modal displacements for forced vibrations mode 1
for t1=1
   ud1=u1*(1/(1-(pF/ppr1)^2))*sin(pF*t1)
end
% Modal displacements for forced vibrations mode 2
for t2=2
   ud2=u2*(1/(1-(pF/ppr2)^2))*sin(pF*t2)
end
% Modal displacements for forced vibrations mode 3
for t3=3
   ud3=u1*(1/(1-(pF/ppr3)^2))*sin(pF*t3)
end
% Total dynamic displacements for forced vibrations
UD=ud1+ud2+ud3
```

Having determined the modal and total displacements values can be expressed the axial sectional efforts using the following relation (Deacu L., 1983; Fetea M., 2010):

$$F_i = \rho \cdot p_i^2 \cdot X_i(x) = \rho \cdot p_i^2 \cdot \frac{(2i-1)\pi}{2l} \cdot x \tag{7}$$

For modal forces static applied at considered times

$$F_1 = \rho \cdot p_1^2 \cdot \frac{(2-1)\pi}{2l} \cdot x$$

$$F_2 = \rho \cdot p_2^2 \cdot \frac{(2 \cdot 2-1)\pi}{2l} \cdot x$$

$$F_3 = \rho \cdot p_3^2 \cdot \frac{(2 \cdot 3-1)\pi}{2l} \cdot x$$

In the principal sections the axials sectionals efforts will be constant and equal with the values forces applied (Fetea M., 2010; Deacu L., 1983).

$$N_1(x) = F_1 = \rho \cdot p_1^2 \cdot \frac{(2-1)\pi}{2l} \cdot x$$

$$N_2(x) = F_2 = \rho \cdot p_2^2 \cdot \frac{(2 \cdot 2-1)\pi}{2l} \cdot x$$

$$N_3(x) = F_3 = \rho \cdot p_3^2 \cdot \frac{(2 \cdot 3-1)\pi}{2l} \cdot x$$

Modal dynamic response will be calculated by multiplying the sectional efforts by dynamically multiplication function (Albu A., 1986; Fetea M., 2010).

$$N_i(x,t) = \rho \cdot p_i^2 \cdot \frac{(2i-1)\pi}{2l} \cdot x \cdot \Psi_i(t) = \rho \cdot p_i^2 \cdot \frac{(2i-1)\pi}{2l} \cdot x \cdot \frac{1}{1-\left(\frac{\omega}{p_i}\right)^2} \sin \omega t_i \quad (8)$$

Having determined the dynamic modal axial efforts, applying the principle of superposition we obtained the value of total the dynamic toat axial effort.

Numerical values determination was done for dynamic sectional efforts using calculation software MATLAB (Morar L., 2006):

```
E=2.1*(10^5) % Young's modulus
d=30 % spindle diameter [mm]
L=30 % length spindle [mm]
A=7.85 % spindle material density
pF=300 % disturbing force pulsation
F0=136.0123 % modulus of harmonic disturbing forces
t1=.1
t2=.5
t3=1
x=0:50:300
% Modal cutting forces
F1st=0.00785*(ppr1)^2*(pi/2*L)*x
F2st=0.00785*(ppr2)^2*(pi/2*L)*x
F3st=0.00785*(ppr3)^2*(pi/2*L)*x
% Dynamic axial efforts
for t1=0.1
   N1DIN=F1st*(1/(1-(pF/ppr3)^2))*sin(pF*t1)
end
for t2=0.5
   N2DIN=F2st*(1/(1-(pF/ppr2)^2))*sin(pF*t2)
end
for t3=1
   N3DIN=F3st*(1/(1-(pF/ppr3)^2))*sin(pF*t3)
end
```

RESULTS AND DISCUSSION

The results obtained by applying numerical method are highlighted in some of the main sections of the spindle.

For the modal and total displacements at free vibrations the results are highlighted in table 1.

Table 1

Modal and total displacements for free vibrations

Sections	0	50	100	150	200	250	300
Mode 1	0	-0.325	0.564	-0.651	0.5645	-0.325	0.0
Mode 2	0	0.437	-0.000	-0.437	0.000	0.437	0.0
Mode 3	0	-0.470	-0.814	0.940	-0.814	-0.470	0.0
Total	0	-0.358	-0.250	-2.029	-0.250	-0.358	0.0

The determined values of modal and total and dynamic axial displacement in the principal cross-sections of spindle are shown in Table 2.

Table 2

Modal and total displacements for forced vibrations

Sections	0	50	100	150	200	250	300
mode 1 ∗	0	-0.265	0.460	-0.531	0.460	-0.265	0
mode 2 ∗	0	-0.142	0.	0.142	0	-0.142	0
mode 3 ∗	0	0.006	-0.011	0.013	-0.011	0.006	0
Total	0	0.006	-0.011	0.013	-0.011	0.006	0

In figure 1 is represented the variation of total displacements of spindle free vibrations and in figure 2 of forced vibrations.

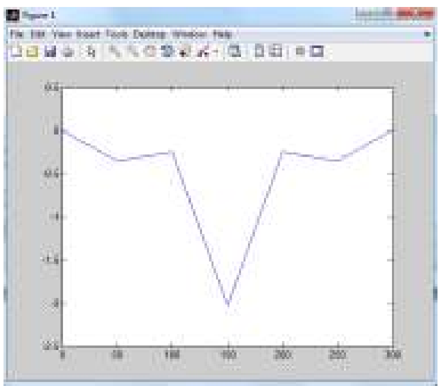

Fig. 1. Total displacemenrs variation

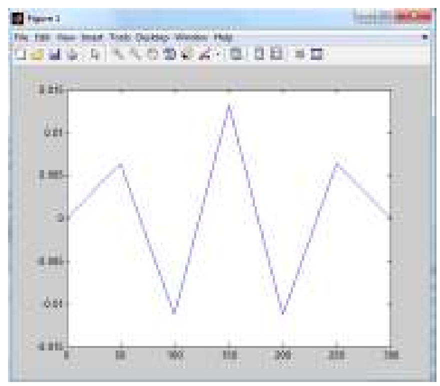

Fig. 2. Total displacemenrs variation

The determined values for modal and total dynamic efforts are presented in Table 3.

Table 3

Modal and total dynamic efforts

Secțiunile	0	50	100	150	200	250	300
mode 1	0	27.8729	55.7458	83.6187	111.4915	139.3644	167.2373
mode 2	0	64.4830	128.9659	193.4489	257.9318	322.4148	386.8977
mode 1	0	705.1	1410.2	2115.3	2820.4	3255.5	4230.5
Total	0	797.4	1594.4	2392.3	3189.8	3987.2	4784.7

CONCLUSIONS

Following the determined values for modal displacements corresponding to free and forced vibration it is found that the spindle having length $l = 300[mm]$, the maximum modal displacement of free vibration are registered in section $x = 150[mm]$ to mode number 3, having value

$u_3 = 0.9407 \cdot 10^{-3}[mm]$, and for forced vibration corresponding to harmonic cutting disruptive force, the maximum modal displacement mode modal movements are registered for mode 1 in the section $x = 150[mm]$, having value $u_3 = 0.5315 \cdot 10^{-2}[mm]$. In percentages is found that the maximum modal displacement obtained by the free vibrations is $\approx 17\%$ of the modal forced displacement.

In table 3 are presented the modal efforts values obtained at dynamic forced vibrations as a result of disruptive force cutting action. The values were obtained by developing a numerical computation using Matlab software. Following some simple analytical calculations it is found the maximum static axial force is recorded for free vibration for mode 3 of vibration in the section $x = 150[mm]$ having value $N_{3static} = 1439[daN]$ and for forced vibrations as a consequence of disruptive force cutting action the maximum effort is registered for mode 3 in section $x = 300[mm]$, having value $N_{3DINAMIC} = 4230{,}5[daN]$. Static axial maximum efforts representing 34% of the maximum dynamic axial effort.

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